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Technical Note

Temperature distributions in an absorbing–emitting–scattering semitransparent slab with variable spatial refractive index

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Abstract

A Monte Carlo curved ray-tracing method is used to analyze the radiative heat transfer in one-dimensional absorbing–emitting–scattering semitransparent slab with variable spatial refractive index. A problem of radiative equilibrium with linear variable spatial refractive index is taken as an example in this paper. The predicted temperature distributions are determined by the proposed method and compared with the data in references. The results show that influences of refractive index gradient are important and the influences increase with the refractive index gradient, the temperature distribution approaches to the one obtained for a constant refractive index when the slab optical thickness is far greater than 1.0, and the effect of the scattering phase function is similar to that in the medium with constant refractive index.

Keywords: Semitransparent medium; Variable spatial refractive index; Monte Carlo method; Curved ray-tracing method

1. Introduction

Recently, the radiative transfer in the medium with variable spatial refractive index evoked the wide interests of many researchers. Ben Abdallah and coworkers [1–5] developed and used a curved ray-tracing technique to analyze the radiative heat transfer in absorbing–emitting semitransparent medium with variable spatial refractive index. Lemonnier and Le Dez [6] applied the discrete ordinates method to solve the radiative transfer cross a slab with variable refractive index. Huang et al. [7,8] and Xia et al. [9] presented a combined curved ray-tracing and pseudo-source adding method for radiative heat transfer in one-dimensional semitransparent medium with graded refractive index. Liu [10] developed a discrete curved ray-tracing method to analyze the radiative transfer in one-dimensional absorbing–emitting semitransparent slab with variable spatial refractive index. Liu and Tan [11] used the discrete curved ray-tracing method to study the transient temperature response in semitransparent variable refractive index medium subjected to a pulse irradiation. All of these works have not taken scattering into account. To our knowledge, no work has investigated radiative transfer in scattering medium with variable refractive index.

Due to have good ability to treat complex boundary geometry and anisotropic scattering, the Monte Carlo method is often used to simulate the radiative transfer in the medium with uniform refractive index. In the processes of the Monte Carlo simulation of radiative transfer, the key problem is the ray tracing. Based on the curved ray-tracing technique developed by Ben Abdallah and coworkers [1–5], the objective of this paper is to develop a Monte Carlo curved ray-tracing method (MCCRT) to analyze the radiative transfer in

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one-dimensional absorbing–emitting–scattering semitransparent slab with variable spatial refractive index, in which the Monte Carlo method is combined with the curved ray-tracing method. A problem of radiative equilibrium with linear variable spatial refractive index is taken as an example to examine the accuracy of the proposed method, and the effects of the refractive index gradient, the slab optical thickness, and the scattering phase function on the temperature distributions are analyzed.

2. Monte Carlo curved ray-tracing method

As shown in Fig. 1, we consider one-dimensional semitransparent gray absorbing–emitting slab with thickness z_L. The boundaries are black walls. The temperatures of boundary wall are imposed as T_0 and T_L, respectively. The absorption coefficient k_a and scattering coefficient k_s are uniform over the slab, but the refractive index n of medium varies linearly with the axis coordinate z.

In the processes of the Monte Carlo simulation of radiative transfer, the key problem is the ray tracing. As shown in Fig. 1(a), if $\theta_1 < \pi/2$, the curvilinear abscissa S on the ray trajectory from z_1 to z_2 can be written as [1]:

$$S = \frac{z_L}{n_L - n_0}\left[\sqrt{n^2(z_2) - n^2(z_1)\sin^2\theta_1} - n(z_1)\cos\theta_1\right], \quad \theta_1 < \frac{\pi}{2}. \tag{1}$$

Similarly, if $\theta_1 > \pi/2$ and $n(z_1)\sin\theta_1 \leqslant n(z_2)$, as shown in Fig. 1(b), the curvilinear abscissa S on the ray trajectory from z_1 to z_2 can be written as

$$S = -\frac{z_L}{n_L - n_0}\left[n(z_1)\cos\theta_1 + \sqrt{n^2(z_2) - n^2(z_1)\sin^2\theta_1}\right], \quad n(z_1)\sin\theta_1 \leqslant n(z_2). \tag{2}$$

If $\theta_1 > \pi/2$ and $n(z_1)\sin\theta_1 = n(z_2)$, the ray is totally reflected back at z_2, and the curvilinear abscissa S on the ray trajectory from z_1 to z_2 can be expressed as

$$S = -\frac{z_L n(z_1)\cos\theta_1}{n_L - n_0}, \quad n(z_1)\sin\theta_1 = n(z_2). \tag{3}$$

Similar to the Monte Carlo simulation of radiative transfer in the medium with uniform refractive index, in the simulation processes of radiative heat transfer in the medium with spatial variable refractive index by MCCRT, the radiant power emitted by volume element i and absorbed by volume or surface element j is written as

$$Q_{ij} = 4\pi k_i\,\Delta V_i n_i^2 I_{bi} D_{ij}, \tag{4}$$

where ΔV is the volume of element. The radiation distribution factor D_{ij} is defined as the fraction of the total radiation emitted from surface or volume element i that is absorbed by surface or volume element j. The radiation distribution factor D_{ij} is determined using MCCRT. Similarly, the radiant power emitted by surface element i and absorbed by volume or surface element j is written as

$$Q_{ij} = \varepsilon_i \pi n_i^2 I_{bi}\,\Delta A_i D_{ij}. \tag{5}$$

After solved D_{ij}, radiant heat absorbed by element i can be calculated by

$$Q_i^a = \sum_{j=1}^{N} 4\pi\,\Delta V_j k_j n_j^2 I_{bj} D_{ji} + \sum_{l=1}^{M} \pi\,\Delta A_l \varepsilon_l n_i^2 I_{bj} D_{li}. \tag{6}$$

3. Results and discussion

To examine the accuracy of the method presented in this paper, a problem of radiative equilibrium inside an absorbing–emitting semitransparent gray slab with linear variable spatial refractive index is taken as an example. The boundaries are black walls, and the boundary temperatures are and imposed as $T_0 = 1000$ K and $T_L = 1500$ K, respectively. The absorption coefficient k_a and scattering coefficient k_s are assumed to be constant within the slab, and the scattering phase function is assumed to be linear as $\Phi = 1 + b\cos\theta$. At radiative equilibrium, the temperature of volume element i is determined by

$$T_i = \left[\frac{Q_i^a}{4k_a\,\Delta V_i \sigma n_i^2}\right]^{0.25}. \tag{7}$$

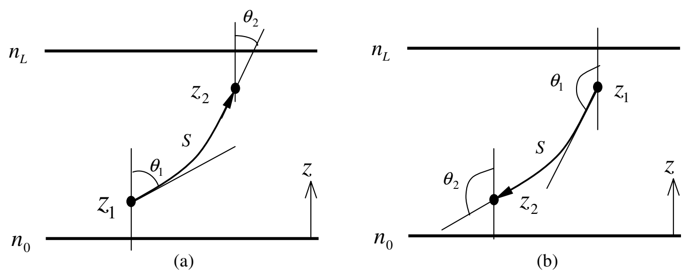

Fig. 1. Physical model and ray tracing: (a) $n(z_1) \leqslant n(z_2)$ and $0 \leqslant \theta \leqslant \pi/2$; (b) $n(z_1)\sin\theta_1 \leqslant n(z_2)$ and $\pi/2 \leqslant \theta \leqslant \pi$.

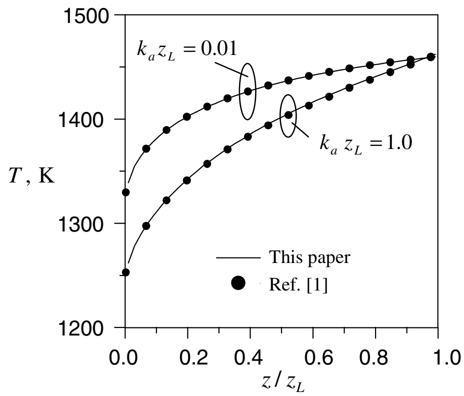

Fig. 2. Temperature distributions in the cases of $n_0 = 1.2$, $n_L = 1.8$ and $\omega = 0.0$.

The temperature distributions in the cases of $n_0 = 1.2$, $n_L = 1.8$ and single scattering albedo $\omega = 0.0$ are calculated. The results are shown in Fig. 2, and compared with the values of Ref. [1]. It can be seen that the results calculated by the MCCRT agree very well with the ones in references.

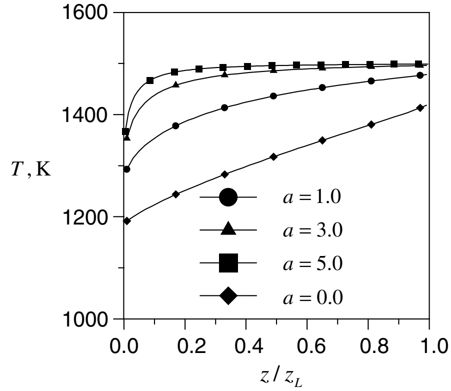

Fig. 3. The effects of refractive index gradient on the temperature distribution in the case of $n_0 = 1.0$, $(k_a + k_s)z_L = 1.0$, single scattering albedo $\omega = 0.2$, and $b = 0.0$.

Fig. 3 shows the effects of refractive index gradient on the temperature distribution in the case of $n_0 = 1.0$, $(k_a + k_s)z_L = 1.0$, $\omega = 0.2$, and $b = 0.0$, in which the refractive index gradient is defined as

$$a = \frac{n_d - n_0}{(k_a + k_s)z_L}. \tag{8}$$

As shown in Fig. 3, the influences of refractive index gradient are important and the influences increase with the gradient a. Under the condition of radiative equilibrium, the medium temperature is controlled by the emissive power of slab boundaries. Because the emissive power of slab boundary at $z = z_L$, $n_L^2 \sigma T^4$, increases with the refractive index gradient, the medium temperature becomes more uniform with the increase of the refractive index gradient. The medium temperature approaches to T_L when the refractive index gradient tends to positive infinite.

Fig. 4 shows the effects of the slab optical thickness on the temperature distribution in the case of $n_0 = 1.2$, $n_L = 1.8$, $\omega = 0.3$, and $b = 0.0$. By comparison with the temperature distribution obtained for a constant refractive index ($a = 0$), it can be seen that, with the increase of the slab optical thickness, the difference of temperature decreases, and the temperature distribution approaches to the one obtained for a constant refractive index ($a = 0$) when the slab optical thickness is far greater than 1.0.

Fig. 5 shows the effects of the scattering phase function on the temperature distribution in the medium with variable spatial refractive index in the case of $n_0 = 1.2$, $n_L = 1.8$, and $(k_a + k_s)z_L = 1.0$. The effect of the scattering phase function is similar to that in the medium with constant refractive index.

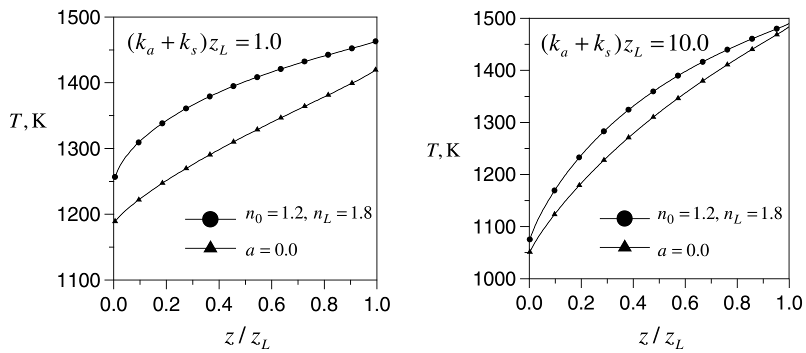

Fig. 4. The effects of the slab optical thickness on the temperature distribution in the case of $n_0 = 1.2$, $n_L = 1.8$, $\omega = 0.3$, and $b = 0.0$.

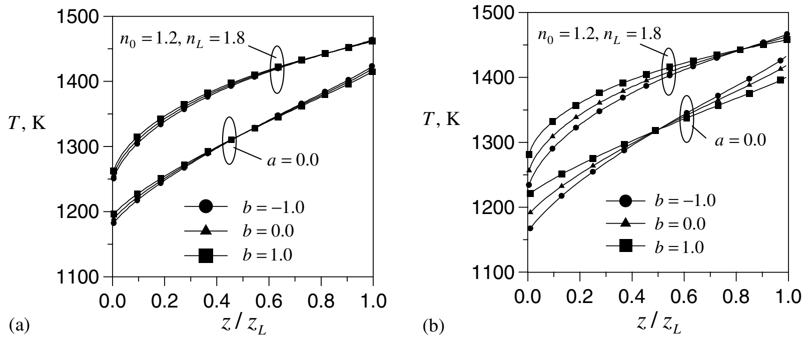

Fig. 5. The effects of the scattering phase function on the temperature distribution in the medium with variable spatial refractive index in the case of $n_0 = 1.2$, $n_L = 1.8$, and $(k_a + k_s)z_L = 1.0$: (a) $\omega = 0.2$ and (b) $\omega = 0.8$.

4. Conclusions

A MCCRT method is used to analyze the radiative transfer in one-dimensional absorbing–emitting–scattering semitransparent slab with variable spatial refractive index. A problem of radiative equilibrium with linear variable spatial refractive index is taken as an example here. The main conclusions can be made as follows:

1. The MCCRT method has a good accuracy in solving the radiative transfer in one-dimensional semitransparent slab with variable spatial refractive index.
2. The influences of refractive index gradient are important and the influences increase with the refractive index gradient.
3. The temperature distribution approaches to the one obtained for a constant refractive index when the slab optical thickness is far greater than 1.0.
4. The effect of the scattering phase function is similar to that in the medium with constant refractive index.

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